Original Article

Task Selection and Implementation: An Account of Mathematics Teachers' Values

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Abstract

Students achieve the goals of learning mathematics through carefully chosen, organized, and implemented tasks. This study examined teachers' arguments as reflections of their values regarding task selection and implementation. Specifically, it aimed to explore teachers' values in selecting mathematical tasks and how they carried out these tasks. Valsiner's Zone theory was employed as a theoretical framework to guide the research from a sociocultural perspective. To fulfil the research objectives, various qualitative data were collected and analysed through instrumental case studies, primarily consisting of value clarification interviews and observations. Narratives from the interviews and observations were analysed using the constant comparison method. The findings revealed that teachers have had distinct approaches when it comes to selecting and implementing mathematical tasks. One of the teachers in the study prioritized conceptual comprehension and emphasized the interconnectedness of mathematical concepts over procedural fluency. Conversely, another teacher focused on student engagement, participation, and practical application of mathematics, promoting active discussions, group work, and critical thinking among students. Therefore, the study underscored the significance of teachers clarifying their values to effectively teach mathematics and enhance student comprehension within the Ethiopian educational framework.

Keywords: Implementation, Mathematical tasks, Teachers' arguments, Selection, Valuing

1. Introduction

In Ethiopian secondary schools, mathematics textbook for each grade is centrally published by the Ministry of Education. Teachers are responsible for using this textbook to choose tasks, arrange order, and prepare tasks for teaching. Especially, when teachers have one common textbook to use, the selection and implementation of tasks demand their decisions and actions that reveal their values. It directs the implementation of tasks towards the realization of what teachers think is worthwhile to enhance students' understanding of mathematics (Seah, 2013). Utilizing carefully selected mathematical tasks in a classroom empowers students to engage in various intellectual undertakings (Bozkurt et al., 2023).

Despite its good articulation in mathematics educational research, policy documents, and practice, values are viewed and defined in different depths and breadth (Bishop, 2020; Sam & Ernest, 1997). The 'definitional inconsistency' and interpretation of values construct in mathematics educational research is epidemic (Hannula, 2012). At 'the heart of the confusion', Rohan (2000) discusses two ways of using it: values as nouns and values as a verb (to value). The theoretical explanation for the use of value as a verb "implies that some higher level evaluation has taken place" (Rohan, 2000, p. 256). In this study, values are considered evaluation functions that are described by constructs akin to guiding principles, standards, and criteria for decisions and actions (Jablonka & Keeitel, 2006). When teachers express they value (to value) certain mathematical tasks and ways of implementation, they are expressing a deeper meaning associated with it (Rohan, 2000). Thus, teachers' arguments for the choice of tasks and their implementations embody their underlying values. Focusing on this aspect, teachers' valuing refers to what they regard as worthwhile for students' learning of mathematics (Seah, 2013). As such, in this study, the uses of values are on the process rather than the underlying values.

Values are fundamental qualities that we consider important influencing our decisions and motivating our actions. They are deeply rooted in our truths and commitments, guiding our long-term choices and priorities (Seah, 2019). As a mathematics teacher, when we think of our values, we consider the principles and beliefs that are fundamental to our teaching practice. Our values in education may include fostering a love for learning, promoting critical thinking skills, encouraging perseverance and problem-solving, and cultivating a supportive and inclusive classroom environment. These values are important to us as educators because they guide our decisions, shape our interactions with students, and ultimately contribute to the academic and personal growth of our students (Schwartz, 2012).

For this study, mathematical tasks are characterized in a wider sense to include activities, exercises, group work, problems, and word problems "with clearly formulated assumptions and questions, known to be solvable in predictable time by students" (Sierpinska, 2004, p. 10). By emphasizing how teachers choose tasks, justify their choices, and implement them in a classroom, a spiral of teaching cycle model is designed to understand the selection and implementation of functions. The model involves planning; teaching and observing; reflecting and analysing; and then re-planning the next lesson based on the teacher's self-reflection (Berg, 2009; Jaworski, 2007). In this study, beyond an attempt to analyse teachers' arguments for the selection and implementation of mathematical tasks, the analysis includes the teachers' directed efforts and the learning environment.

The focus of this study was on analysing the teachers' arguments for selecting and implementing mathematical tasks as an expression of their value system. Specifically, it dealt with how teachers select tasks, justify their choices, and implement tasks in the classroom. In this regard, the valuations of tasks that are related to what they promote and allow in the classroom are emphasized. In general, the study aimed to explore teachers' selection and implementation of mathematical tasks to understand values expressed by teachers' arguments.

Accordingly, the following research questions were designed: *how do teachers justify their values in selecting and implementing mathematical tasks?* And, *how do teachers set up and implement their chosen tasks?* In the first research question, teachers' justification for the selection, modification, and introduction of new mathematical tasks and the implementation of these tasks from available resources infer teachers' values. The argument refers to the teachers' justification, the process of reasoning, or the situation they use to explain why or why not they choose mathematical tasks from the textbook. In the second research question, the concept of 'setup' refers to the organization and arrangement of tasks. In a broader sense, it includes the methods by which teachers introduce the task, or "the teachers' communication with students regarding what they are expected to do, how they are expected to do it, and with what resources" (Stein et al., 2000, p. 25). The implementation of tasks refers to the process where the teacher and the students start to work on mathematical tasks (Stein et al., 2000).

2. Theoretical framework

Drawing on Vygotsky's notion of the zone of proximal development (ZPD) and Lewin's Zone of free movement (ZFM), Valsiner developed his Zone theory. Valine's Zone Theory provides a framework for understanding teachers' values within a sociocultural context by emphasizing the social setting, goals, and actions of participants. It allows for the exploration of how teachers' values influence their decision-making processes in selecting and implementing mathematical tasks. The theory highlights the dynamic interaction between teachers and students, as well as the role of the teacher in promoting certain actions and guiding students' learning. By applying the Zone Theory, this study gains insight into how teachers' values shape their teaching practices and ultimately impact student learning outcomes within the Ethiopian educational framework.

VZT employs additional aspects of social relations to ZPD: that are “the social setting and the goals and action of the participants” (Valsiner, 1997). As he is interested in the whole set of possibilities that may or may not be actualized, his advancement of ZPD focuses on micro-genetic studies that give attention to the immediate processes of emerging new phenomena (Galligan, 2008). In classroom learning, the usefulness of ZPD is dependent on other enabling conditions of the learning environment such as the nature of mathematical tasks and the setting (social and physical). Valsiner introduced two additional Zones - the zone of free movement (ZFM) and the zone of promoted action (ZPA).

The ZFM is the “socially constructed cognitive structure of child-environment relationships” (Valsiner, 1997, p. 189). It describes the possibility of different actions at any given time in a particular environment. Freedom of actions and thoughts within this zone is non-restricted; however, activity outside the bounded area is not permitted. Thus, it structures the environment of the students in terms of access to the area, availability of different objects, and the way the student acts. It plays a significant role in structuring the current and future actions of the child in a given environment. It characterizes “the set of what is available (in terms of areas of environment, objects in those areas, and ways of acting on these objects) to the child’s acting in the particular environmental setting at a given time” (Valsiner, 1997, p. 317). ZPA is “the set of activities, objects, or areas in the environment, in which the person’s actions are promoted” (Valsiner, 1997, p. 192). It “illustrates the direct efforts of the people around the child to guide his or her actions in one, rather than another, direction” (Valsiner, 1997, p. 317). The actions could be promoted through discussion, in the tools made available to students, and by example. It refers to the efforts of a teacher, the curriculum, or others to promote actions. However, as ZPA is not binding, students may comply with or reject what is promoted by their teachers or curriculum. The teacher by encouraging may guide the students to go beyond the existing boundaries of the ZFM. ZPA can also restructure the ZFM.

In the context of the present study, ZPA is a set of tasks offered by the teacher that are oriented toward the promotion of new skills. Since ZPA describes what teachers promote in the classroom, any teachers' actions aimed towards the students' learning of mathematical concepts, such as teachers' approaches to problem-solving, guiding students' work, giving examples, and others are all part of the ZPA. Though the teacher is not the sole determinant, in the actual classroom teaching, it is her/his responsibility to select, rearrange, and implement tasks for classroom learning. Thus, she/he decides what is worthwhile for students' understanding of mathematics. These choices guide what action(s) to be promoted in the classroom. ZFM defines what the teacher values and allows from what is available in the classroom. ZFM and ZPA interact and work "jointly as the mechanisms by which canalization of children's development are organized" (Valsiner, 1997, p. 110). Valsiner argues that these zones work together as a ZFM/ZPA complex. The ZFM/ZPA as complex, dynamic, and interrelated is constantly reorganized by the teacher through interactions with students. As such, they are "always temporary, constantly changing structures that organize the immediate construction of the future state out of a here-and-now setting" (Valsiner, 1997, p. 319).

Valsiner's main concern is the development of children concerning guiding others and the environment. However, Goos (2009) argues that Zone theory can also be interpreted and used to analyse teachers' development (Focus on Teacher-as-Teacher, Teacher-as-Learner, and Teacher-Educator-as-Learner). My interpretation of zone theory is from the perspective of the teachers' choice of tasks and their implementation in the classroom. Therefore, the ZFM/ZPA complex as an interacting space between the teacher and the students was used to explore how teachers' valued tasks are implemented in the classroom.

3. Methods

The study was an instrumental case study design employing a qualitative approach. During the research, two teachers from a public school took part in the study. They were teaching exponential and logarithmic functions to tenth-grade students. Data were gathered using tape-recorded value clarification semi-structured interviews and video-recorded classroom observation. All data were transcribed for analysis. The main concern of the value clarification interviews (Raths et al. (1987) during the selection of tasks was to clarify the difference between value indicators and value. The interview assisted the teachers in explicating their arguments and reasons that involve valuing processes for choosing or not choosing particular tasks within the realms of task selection. The video-recorded lesson observations were related to the implementation of tasks within the realm of acting. The constant comparison method from grounded theory complemented the analysis of the study. It was used to compare: (a) Data within the interview: this comparison was made within each interview to identify, compare, and contrast among the meaningful elements concerning teachers' valuing. (b) Comparison between data from interviews and data from classroom observations. This level helped to compare what was chosen and prized by the teachers against their actions.

After identifying the value indicators, interview data were coded and compared with existing codes and categories (Cohen et al., 2011).Three levels of coding: open coding, axial coding, and selective coding were used as tools for the analysis of the data. In the initial stages, value indicators as meaningful elements were identified from the interviews. Then, in the axial coding stage, the interview data which was mainly the teachers' argument for choosing tasks were cross-checked with other data sources such as video-recorded classroom teaching. It was coded and categorized to refine the open coding. Finally, with selective coding, the core codes were identified to form the 'storyline'(Creswell, 2013).

4. Results

The following two sections summarize the cross-analysis of the data to see similarities and contrasts. The presentation of the cross-case analysis addresses issues about teachers' values and task choice and implementation. The first section focuses on teachers' values about their task selection of tasks while the second one deals with teachers' values vis-à-vis implementation of tasks. In each section, findings of data gathered from the first teacher, Degu, and the other one, Agar will be presented respectively. The findings were informed by the teaching cycle in a way of addressing the two research questions. In this regard, teachers' values and the way they set up and implemented tasks are presented.

The rationale behind teachers' values in selecting and implementing mathematical tasks

Degu's values structure: Valuing conceptual understanding

During the first interview that dealt with his background and experience, he emphasized that mathematics should be taught for *conceptual understanding*. His explanation of teaching for *conceptual understanding* is that students learn mathematics through structure and relation that has evolved from simple understanding to more complex abstraction. For him, understanding concepts and their relation is the basis for learning mathematics. In the initial process of the analysis, three categories for his justification of task selection and implementation are identified: *conceptual understanding, connectedness of mathematical concepts,* and *uniqueness of tasks*. In this article, how valuing *conceptual understanding* is related to the ways he presented exponential and logarithm functions to his students is discussed.

One of the interview questions that clarifies teachers' values is related to the identification of the indicators based on the Raths et al. (1987) category that focuses on teacher's aspirations. The teacher was asked what he thinks is important when teaching mathematics.

51. *Interviewer: What is the most important point you want your students to grasp when learning mathematics?*
52. *Teacher: (...) When I teach, I prioritize focusing on the concept. I ensure that students understand the specific content before moving on to other exercises. Understanding the concept is key before delving into further practice.*
53. *Interviewer: Why do you think teaching concepts is important in mathematics?*
54. *Teacher: Unless they [students] are clear with the concept, they cannot do other things. Students should first grasp the concept. I believe, that after grasping the main concept, students will not be in trouble to solve the forthcoming problems. I am saying this from my own experience. If I am not clear with the concept, then it will be exceedingly difficult for me to go further.*

Degu interview I, 6.11.2012.

In the above extract, Degu responded to the clarifying interview question (52) that teaching mathematics for *conceptual understanding* is fundamental in his teaching. To further clarify the reason why he focused on concepts, he mentioned that if students learn the concept, then it will enhance their understanding and make them apply the concept they understand to other similar or related situations easily (54). He also mentioned the importance of considering one's own experience. His choice resulted from his thoughtful consideration of the consequences. Degu claimed that it is important for students to understand the basic concept of the specific content before they engage in solving mathematical problems. For him, classroom lessons should start by providing students with the basic concept of the content and then move to problems or exercises. Focusing on *conceptual understanding* is one of Degu's arguments in his task selection. For him, application or any other extensions should not be interwoven with the basic concepts. The teaching of concepts should come first. He argued that mixing them with concepts constrains students' understanding of the basics. He metaphorically described the concept as a 'key' for understanding.

For instance, when he taught the graph of logarithmic functions, he argued that by teaching all the general properties of the function $Y = \log_2 x$, it is possible to help students draw and state other similar logarithmic functions.

106. *Teacher: Next week, I will teach the basic concepts first. Then, students will be able to identify the nature of the graph without drawing it. They will confidently describe the characteristics and shapes of the graph by visual inspection.*

107. Interviewer: Do you think that the lesson you planned will help them identify the nature of graphs?

108. Teacher: Of course!

109. Interviewer: Do you mean without sketching?

110. Teacher: Yes, after I address my lesson and help them to grasp the main concept.

111. Interviewer: Could you tell me how?

112. Teacher: For instance, I will choose the logarithm x to the base ten and one over ten to teach the whole concept of drawing the graphs. Once they understand these two graphs, then they can do others by themselves.

Degu interview II, 8.11.2012

Degu claimed that after he introduces students to 'the basic concept' (106, 110), students might apply and visualize the nature of different graphs of logarithmic functions. Degu's ways of presenting graphs of a function are at a conceptual level. He argued that if students understand the basic property of the function, they may state the property of graphs without sketching it. This is also observed during classroom observation. The other point he mentioned concerning *conceptual understanding* is related to the way he presented the tasks in the classroom. During the third interview, he reflected on his previous lessons.

223. Interviewer: How was your previous lesson?

224. Teacher: *As for me the main thing is that … I always get into the class to let them know the very concept. I think I try to use my utmost effort to do that. There should not be anything that the students miss. I will evaluate whether they grasped the concept or not. I give them some tasks and when we do them together, I can see where the problem is. It is on Monday that we will discuss these tasks. I will not start a new topic before that.*

Degu interview II, 8.11.2012

He assessed his effectiveness in teaching by considering the time he dedicated to mathematical concepts. Accordingly, he designed his classroom lessons in a way of showing them every detail. In the above extract (224), Degu justified his attainment of the objectives in terms of his devotion to showing/giving detailed information to his students. The nature of the mathematical environment was characterized by the way he organized the lesson, the promoted tasks, and the zone of free movements. Students' engagement was restricted to careful attention to the teacher's explanation. He mentioned that "there should not be anything that the students miss."

Agar's value structure: Valuing learners' participation

Agar's value structure was initially identified through indicators such as *valuing classroom participation, application of mathematics in real-life situations, connectedness to daily life, and lecturing*. These indicators were then compared against seven criteria from Raths et al. (1987). Agar emphasized the importance of classroom participation in her teaching influenced by her own experiences as a student in both church and formal schools. She believed that students' engagement and discussion were crucial for their learning objectives.

Agar's past experiences and educational background appeared to shape her preference for classroom participation as a key element in teaching and learning mathematics. This understanding led her to prioritize student engagement in her mathematics teaching, which was evident in my observations of her classroom where students seemed to have more opportunities for active participation compared to other classes. Like Degu, Agar also acknowledged the influence of her high school geography teacher on her teaching approach.

During the interview, Agar emphasized the importance of student participation in learning mathematics and stressed that discussions should align with the learning objectives. She believed that student involvement in the classroom should be aimed at helping them achieve their learning goals. I interpret her view on classroom participation as a purposeful action, like what Habermas (1984) describes as teleological action, where decisions are made based on a rational interpretation of the situation to achieve specific objectives. Through interviews with the teacher, it became clear that Agar engaged in discussions with students to determine how to best manage the classroom environment. Agar with her students considered various options, such as maintaining the current approach or implementing new strategies to meet the learning objectives, based on students understanding and interpretation of the learning environment.

According to Wertsch (1991), the sociocultural approach of a goal-oriented action employed a meditational means. Here, the teaching of mathematics in this school is mediated by the school curriculum, the teacher planning, and the way the tasks are designed. But it is also mediated by the rules set by the teacher and the students after their decision. These rules include explicit regulation to set classroom learning environments that allow students to participate. Valsiner (2007) argues that goal-oriented human action and the different orientations are culturally valued.

Two main criteria guided Agar's task selection process are: the first criterion focused on the extent to which tasks facilitate group discussions among students. She noted that the textbook offered a variety of tasks that encourage group work, where students collaborate to solve problems. The second criterion she considered was the difficulty level of the tasks. Agar preferred to challenge her students with complex questions that push them to think critically and solve problems independently.

Furthermore, Agar's task selection was influenced by her emphasis on student participation in the classroom. As a result, she chose tasks that promoted active discussion among students. In contrast to Degu's approach, she ensured that all tasks provided in the textbook were utilized for classroom discussions. The following excerpt from an interview highlights the types of tasks Agar selected for her teaching:

> *I emphasize activities, group work, and challenging questions from the exercises. The students are familiar with my teaching style. I encourage them to engage in group discussions, regardless of whether they have grasped the concept yet. By allowing them to discuss with their peers first, they can better articulate their thoughts. Therefore, I always prioritize group activities and discussions. Agar interview II, 9.11.2012*

This was also confirmed by one of the students during the focus group discussion. The student mentioned:

> *The most valuable aspect of my mathematics teacher's approach is the incorporation of group work and preparatory activities at the start of each chapter. This was notably absent in my previous teachers' methods. By encouraging us to engage in discussions and exercises, she provides us with a comprehensive understanding of the material. This practice is particularly beneficial for our Matric exam. FGD, 10-09, 15.11.2012*

This student's assertion was further supported by the interview with another teacher, Degu, who took part in the research. Degu mentioned that he was hesitant to allocate time for group work and additional activities due to time limitations.

Setting up and implementation of selected tasks

The case of Degu: Draw the graphs of logarithmic functions

In the following section, selected episodes from classroom task implementation are presented. The selection of those episodes for analysis was made after observing all the videos. The assumption and the criterion for choosing these videos are based on their relatedness and potential to address the research issues.

The context of this lesson was that students had already learned how to draw the graph of an exponential function with different bases. The teacher also assumed that students could recall what they knew about the graph of an exponential function with a base e. Students were introduced to the number e and the graph of $f(x) = e^x$ two weeks ago. Though, the focus of the lesson in the teacher's plan was on how to draw the graphs of logarithmic function $y = \log_b x, where\ b > 1$, the teacher decided to go beyond the objective of the lesson i.e., revising the graph of an exponential function with base e. The teacher restructured his planning to include a topic that was already discussed in the previous lesson. Though he planned to deal with how to draw a logarithm function, he decided to broaden the existing boundaries of the ZFM and tried to relate exponential and logarithm graphs.

The teacher believed that by expanding the subject matter for the day, he would give the students access to a broader perspective. The teacher promoted a discussion on the concept and graphs of the exponential function with base e. Here, it is evident that the teacher's promoted action was not binding. Students might not actively participate in the discussion or lose their interest. The teacher's promoted action restructured students' ZFM by helping them to recall what they had learned previously. Since the teacher also viewed mathematics as an interrelated concept, he decided to revise the previous lesson so that students could link the previous lesson with the new one.

The textbook introduced how to calculate continuous compound interest using the concept of numbers e. It began the discussion with an example of a case from the bank system:

"Annual rate $r = 100\% = 1$, $i = \frac{1}{n}$, if there are *n* periods of compounding per year, then the amount after one year is given by the formula equals $f(x) = (1 + \frac{1}{n})^n$. However, the teacher decided to teach without addressing the issue of banking. And he introduced the number e from its conceptual point of view. He argued that mathematical concepts should come first. Combining concepts with their application makes the task more difficult for the student. The teacher restructured the ZFM/ZPA based on what he thought was worthwhile for the student's learning. Though the application of the number e was promoted by the textbook, the teacher decided to narrow the students' zone of free movement.

After introducing the numbers e, the teacher continued to teach the students how to draw the graphs of logarithmic functions. Degu expanded the rules for students by reintroducing exponential functions, thus broadening their exposure to different mathematical areas beyond the initial scope. While the initial focus was on drawing logarithmic functions, Degu allowed students to explore the relationship between exponential and logarithmic graphs. Initially, he guided students in graphing the logarithm function equal to $f(x) = \log_2 x$ using table construction, and the general properties of logarithms. However, upon noticing confusion between logarithmic and exponential graphs, Degu encouraged students to apply the concept of inverse functions to plot logarithmic graphs. He facilitated additional actions by prompting students to recall previous lessons and connect them to the current topic and reintroduced the concept of transforming quadratic function graphs to aid in graphing logarithmic functions.

Unlike other teachers, Degu empowered students to broaden their understanding and enhance the lesson by allowing them to utilize diverse methods in graphing logarithmic functions. The following diagrammatic illustration shows the multiple representations of logarithmic functions as implemented by Degu.

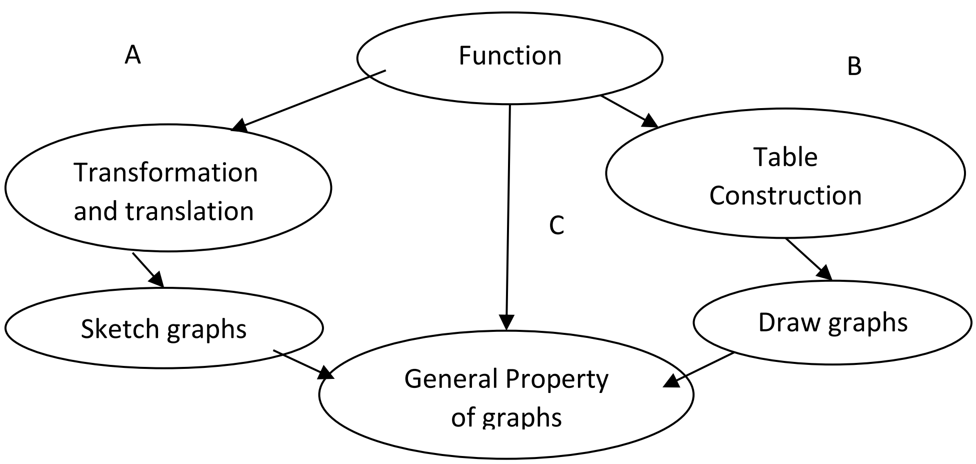

Figure 1: Degu's way of drawing exponential graphs

This task had the objective of enabling students to state the general properties of exponential and logarithmic functions. At the beginning of his lesson, Degu started the lesson by drawing the graphs of the logarithmic function $y = \log_2 x$ using table construction and asked students to state the general property of the logarithm. However, after realizing that students were confusing the graphs of logarithm with exponential function, he allowed the students to use the concept of inverse function to draw the graphs of logarithmic function. The teacher promoted additional action by restructuring students' ZFM by helping them recall what they have learned previously and relate it to the current topic. The focus of the textbook is on the properties of graphs of logarithmic function. Nevertheless, he decided first to re-introduce the concept of transformation of graphs of quadratic function.

As it is shown in the diagram above, Degu presented the lesson in three ways.

1) He introduced a new concept not included in the textbook.

2) He asked students to construct a table and draw the graph then state the property of the graph and

3) He asked students to state the property of the graph directly from the way the function is written.

The case of Agar: Draw the graphs of logarithmic functions.

This section will explore the implementation of a specific task chosen based on its relevance to the research topics. The lesson focused on the graphing of logarithmic functions, where students were tasked with converting an exponential function to a logarithmic form and plotting its graph. The teacher observed the class to ensure students had their exercise books, were in uniform, and were seated properly. The lesson commenced with the teacher prompting students to recall information from the previous class. Following contributions from two students, the teacher summarized their input and proceeded with the day's lesson.

Part of the transcription of the lesson is presented below. Students in the following transcription are indicated as 'St' followed by a number. The number is used to indicate the interaction of the same or different student.

1. T.: What can we use to draw the graph of a logarithmic function?
2. St.: We can use a table.
3. T.: Yes, I told you that we can construct a table. Ok, what type of function is $f(x) = 2^x$?
4. St.: exponential function
5. T.: and $f(x) = \log_2 x$ this?

6. St.: Logarithmic function
7. T.: Ok, now we will together fill the table of the exponential function and you will do the logarithmic function by yourself.

The teacher filled the table by asking the students the values of the function by inserting x into the exponential function. After doing the exponential function table, then the teacher drew the following table and asked the students to do it by themselves.

X							
$f(x) = \log_2 x$	-3	-2	-1	0	1	2	3

The teacher toured around to check what the students were doing. Then the teacher asked if any of them wanted to do it on the chalkboard

8. St1.: $2^x = 2^{-3} = \frac{1}{8}$wrote on the board
9. T.: Is there any of you who got the same answer?
10. St2.: Yes, me
11. T.: First, raise your hand and tell us loudly how you can do it.
12. St2.: I did it like him
13. T.: How do you insert the value of the function in the variable? Look you got different values for the value of the function -3 and 1/8. Of course, the right answer is 1/8. How come this to be true at a time? Look -3, -2, and -1 are values of y but you are substituting for x. ok, any one of you?
14. St3.: y is equal to 2 the power x, and x is…
15. T.: Come and do it on the board
16. St3.:$f(x) = \log_2 x$, $y = 2^x$, $-3 = 2^x$, $x = 2^{-3}$, $x = \frac{1}{8}$
17. T.: Alright, this is an improvement, but the issue remains that you are replacing x with y. Let's examine how it should appear and what actions we can take. Please ensure to adhere to the steps provided. You saw the answer is the same as

$$f(x) = \log_2 x, \quad -3 = 2^x, \text{then} \quad x = \frac{1}{8}$$

Then the teacher filled the rest of the table by asking the students. After completing the table, the teacher then asked the students to show the process on the table, and one of the students did it correctly. After that, the teacher checked the students to make sure that all students understood how to find the value of the function by inserting the value of x in the proper place. The teacher asked, "*Have you all understood how to do it*?". Then the students in the class responded together with a "yes".

Agar's way of dealing with the same topic was different. She started her lesson by drawing the graph of an exponential function $f(x) = 2^x$ and logarithm function $f(x) = \log_2 x$. When she observed some of the students who were wrongly inserting the values of x in $f(x)$, she was interested in making clear how to fill the tables using values of x and how to find the corresponding values of y by inserting values of x. She decided to focus on one way of drawing logarithmic functions table construction, graphing then state general properties. Thus, she asked students to use only table construction methods, and the students were not allowed to use calculators. Though, calculators were used during classroom teaching, they were forbidden during the assessments. When Agar realized that students were not capable of finding the corresponding values of x and y in the equation $f(x) = \log_2 x$. she stuck to the table construction method for graphing logarithmic functions.

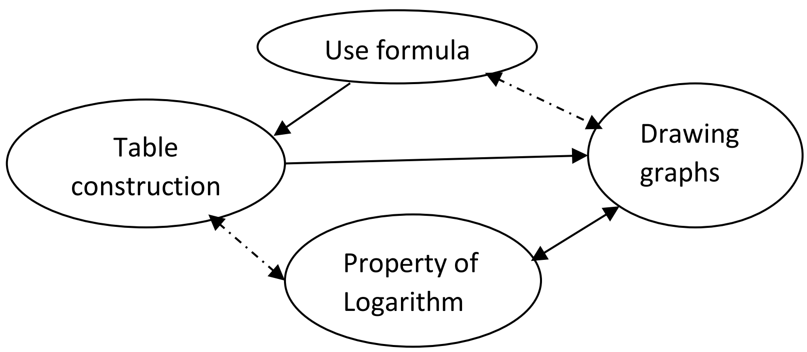

Figure 2: Agar's way of drawing exponential graphs

5. Discussion

Values clarifying interviews, video-recorded observations, and focus group discussions with students were the main instruments to identify teachers' values related to task selection and implementation. Values that were directly related to tasks were analysed in the previous sections. These are *conceptual understanding*, *interrelatedness*, *uniqueness*, and *classroom participation*. Based on the seven criteria stated in (Raths et al., 1987) which are categorized into three: choosing, prizing, and acting four out of nine values in relation to Degue's and Agar's task selection and implementation were identified. Degu emphasized teaching mathematics for conceptual understanding. Degu believed that students should grasp the basic concepts of mathematical content before moving on to problem-solving. He valued conceptual understanding as the foundation for learning mathematics arguing that it enhances students' ability to apply concepts to different situations.

Degu's teaching methods prioritized teaching concepts first before delving into applications or extensions as he believed this approach aids in students' comprehension and retention. His approach involved presenting tasks in a detailed manner and ensuring students grasp the main concepts before proceeding further. Degu's teaching philosophy highlighted the importance of conceptual understanding in mathematics education and its impact on students' learning outcomes. In contrast, Agar's value structure emphasized the importance of learners' participation in mathematics classrooms. Agar's teaching approach was shaped by her personal experiences as a student and her conviction that student engagement and dialogue are essential for successful learning. She emphasized active student participation in classroom tasks and group conversations, to integrate these exchanges with the educational goals. Agar's method of selecting tasks was directed by standards that support collaborative work and present students with intricate queries, fostering critical thinking and problem-solving abilities. In contrast to Degu, Agar guaranteed that every task outlined in the textbook is incorporated into classroom discussions. The research delves into a comprehensive examination of a particular task selected by Agar, centering on graphing logarithmic functions and the subsequent classroom dynamics. The study underscores Agar's commitment to nurturing an interactive learning setting and the beneficial influence it wields on student involvement and academic achievements. The table below showcases the values explicitly stated by the teachers, utilizing Raths's model of value identification.

Table 1: Teacher's value indicators

Values indicators	Degu			Agar		
	Choose	Prize	Act	Choose	Prize	Act
Focusing on conceptual understanding	X	X	X			
Relatedness to other mathematical concepts	X	X	X			
Uniqueness	X	X	X			
Helping students			X			
Multiple representations			X			
Valuing classroom participation			X	X	X	X
Application of mathematics in other fields			X	X		
Connectedness to daily life				X		
Lecturing			X	X		X

The table above indicates that certain value indicators were not classified as values due to not meeting all criteria. In the interview, Degu emphasized the significance of classroom participation in the teaching and learning of mathematics. However, the observation of Degu's classroom did not demonstrate an interactive environment. He attributed this to students' lack of interest and background knowledge. Degu asserted that students must grasp the concepts before engaging in classroom interactions, hence he leaned towards lecturing. Additionally, feedback from a focus group with students revealed some discontent with the teaching approach. Conversely, Degu placed value on assisting students during his free time. Throughout my observation, I witnessed Degu supporting students who encountered challenges in their learning of mathematics.

Agar, the other teacher, emphasized the importance of applying mathematics in daily life for a better understanding of the subject. However, she noted that students may not be prepared to connect mathematics to their everyday experiences. Therefore, she did not consider relating mathematics to daily life as a core value. In my view, the classroom exchange was influenced by the local norms established during previous discussions. Both students and the teacher collaborated to create a supportive learning environment that encouraged active participation in solving well-structured tasks. The interplay of classroom dynamics, teacher objectives, and student values significantly impacted the learning and growth process. The teacher's focus on the learning journey rather than solely correct answers, and the emphasis on involving students in problem-solving activities, exemplified this approach.

Another critical factor was the task presentation and the conducive conditions provided. The task design encouraged students to focus on the process rather than solely on the result, leading to diverse interpretations and conceptualizations of the content. The task was appropriately challenging, falling within the students' Zone of Proximal Development (ZPD), and the supportive learning environment enhanced the chances of successful learning outcomes. The ZPD, as illustrated in the analysis, functioned as a symbolic space where students' mathematical comprehension progressed under the teacher's guidance. The teacher adeptly steered the students towards grasping concepts related to exponential and logarithmic functions, in line with Vygotsky's ZPD theory emphasizing the importance of guidance in improving problem-solving abilities.

Teachers' values vis-à-vis implementation of tasks: Property of graphs of a function as a lens

In this section of the article, a discussion on how teachers set up and implement their tasks will be presented. From Valsiner's zone theory perspective, the discussion on how teachers set up and implement their tasks can be understood in terms of the dynamic interaction between the individual student and the social environment. Valsiner's theory emphasizes the importance of the sociocultural context in shaping individuals' development and learning processes. In this case, the two identified approaches to how teachers present their lessons reflect different ways of structuring the learning environment to support students' engagement and understanding of mathematical concepts.

The first approach, where teachers establish a set of rules that guide their actions, can be seen as a way of creating a structured framework that provides students with clear expectations and boundaries. This structured approach may help students feel more secure and confident in their learning, as they know what is expected of them and how to navigate the learning process.

On the other hand, the second approach, where teachers promote tasks that expand the zone of freedom for students, allows for more flexibility and autonomy in learning. By giving students more freedom to explore and interact with mathematical concepts in their way, teachers can create opportunities for deeper engagement and creative problem-solving.

Overall, both approaches have their merits and can be effective in supporting students' learning in mathematics. By balancing structure and freedom in the learning environment, teachers can help students develop a deeper understanding of mathematical concepts and build the skills they need to succeed in their studies.

Agar in her class, in contrast to another class observed (e.g., Degu's class), where students were limited to a single method of graphing functions, this teacher provided opportunities for students to make mistakes, learn from them, and encouraged active participation. By allowing students to freely explore and make errors, the teacher aimed to foster a supportive and encouraging learning environment. The teacher's approach of posing various questions and promoting student involvement aimed to expand students' ZPD and encourage active engagement. Despite the voluntary nature of participation in the ZPD, it was evident that students utilized their autonomy to engage in the learning process.

By applying Valsiner’s Zone theory, the classroom videos were also analysed to address the second research question about how the teacher set up and implemented tasks. The approach to Valsiner’s Zone theory assumes that:

- ZFM/ZPA complex is defined from the perspective of the teacher as a teacher promoting tasks for the learners within the zone of free movement.

- Teachers with what they are valuing promote tasks that are within the zone of free movement. Thus, what is promoted, theoretically, should be within what is allowed (Blanton et al., 2005).
- ZFM/ZPA continually re-structured teachers' task implementation through teacher and student interaction.

First, it is important to notice that the ZFM/ZPA complex is dynamic. Therefore, the nature of the interaction between the two zones may vary even in a single classroom. In particular, the complex is defined by how the teacher uses the classroom environment to implement tasks and teach the content and what he allows and promotes during the lesson. From the analysis of the data, we came up with two ways of interaction between ZFM/ZPA. The ZFM is a binding agency by which Degu constrains the students' actions in terms of access to areas, objects, or ways of acting on such objects. Also, by broadening and limiting the ZFM the teacher helped students learn mathematics. For instance, Degu, when teaching how to draw the logarithmic function, expanded the ZFM for the students by re-introducing transformation in quadratic function graphs and broadened students' access to different areas which was not primarily promoted. Even though he planned to deal with how to draw a logarithm function, he decided to broaden the existing boundaries of the ZFM and tried to relate exponential and logarithm graphs. In general, the teacher's valuing of tasks and ways of presenting them structured the ZPA and the ZFM. It played a role in affording students access to act upon their learning and in narrowing free movement in each environment. The study implies how teachers may clarify their values to achieve students' understanding through effective mathematics teaching. This study has also theoretical and methodological advances in researching values studies in mathematics education.

6. Conclusion

In this article, the aim is to present teachers' arguments as reflections of their values regarding task selection and implementation. Raths value clarification and Valsiner's Zone theory have provided a tool for elucidating the role of teacher's arguments for choosing tasks. Based on the application of Valsiner's Zone theory in analysing the teachers' task selection and implementation, it can be concluded that the distinct teaching approaches of both teachers reflect their values and beliefs regarding task selection and implementation. One teacher's emphasis on conceptual understanding, student autonomy, and fostering a supportive learning environment aligns with promoting tasks within the zone of free movement. In contrast, the other teacher's structured and directive approach is indicative of tasks that are more controlled and guided, possibly falling outside the zone of free movement.

The comparison between the two teachers' teaching styles not only highlights the diversity of values-based instructional approaches but also underscores the influence of Valsiner's Zone theory on teachers' task implementation. The theory's assumption that teachers promote tasks within the zone of free movement is evident in the contrasting approaches of the two teachers in the study. The first teacher's exploratory approach, focusing on conceptual comprehension and interconnectedness of mathematical concepts, aligns with the idea of promoting freedom of actions and thoughts within the ZFM. On the other hand, the second teacher's structured method, emphasizing student engagement, participation, and practical application of mathematics, reflects a more directed effort to guide students' actions in a specific direction within the ZFM. The application of VZT helps to understand how teachers' values influence their task selection and implementation strategies within the sociocultural context of the classroom. The study emphasizes the importance of considering teachers' values and beliefs in task selection, as they continually re-structure task implementation through teacher-student interaction within the classroom.

Disclose Conflicts of Interest

This paper is free of conflict of interest.

Authors' Contribution

All authors contribute equally.

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